**Title: Dynamics of life expectancy and lifespan equality**

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**Classification:** Social sciences - Demography

**Abstract:**

As people live longer, ages at death are becoming more similar. This dual advance over the last two centuries is a major achievement of modern civilization and a central aim of public health policies. However, some recent exceptions to the joint rise of life expectancy at birth and lifespan equality make it difficult to determine the underlying causes of this relationship. In this article, we develop a unifying framework to study life expectancy and lifespan equality over time relying on concepts about the pace and shape of aging. We study the dynamic relationship between life expectancy and lifespan equality with reliable data from the Human Mortality Database for 49 countries and regions. Our results demonstrate that both changes in life expectancy and lifespan equality are weighted totals of rates of progress in reducing mortality. This finding holds for three different measures of the variability of lifespans. These weights evolve over time and indicate the ages at which reductions in mortality increase life expectancy and lifespan equality: the more progress at the youngest ages, the tighter the relationship. The link between life expectancy and lifespan equality is especially strong when life expectancy is less than 70 years. In recent decades, however, life expectancy and lifespan equality have occasionally moved in opposite directions due to larger improvements in mortality at older ages or a slowdown in midlife mortality. Saving lives at ages below life expectancy is the key to increase both life expectancy and lifespan equality.

**Significance Statement:**

How life expectancy and lifespan equality have evolved over time and why they are so tightly linked are major questions that have triggered considerable scientific interest. Lower lifespan equality implies greater uncertainty in the timing of death length of life for individuals and larger heterogeneity at the population level, and can be a manifestation of social disparities. We show how patterns of change in life expectancy and lifespan equality can be described by weighted age-specific trajectories of mortality improvements over time. The strength of the relationship between life expectancy and lifespan equality is not coincidental but rather a result of progress in saving lives at different ages: the more lives saved at the youngest ages, the stronger the relationship is.

**Main text:**

The rise of human life expectancy over the past two centuries is a remarkable achievement of modern civilization (1, 2). At present, Japanese women have the highest national life expectancy at birth above 87 years. In 1840, that record was held by Swedish women, with an average lifespan of 46 years (3). This exceptional progress has been accompanied by an increase in lifespan equality: in low mortality populations today, most individuals survive to similar ages (4-9). Lifespan equality matters because it captures a fundamental type of inequality: variation in lengths of life. This variation is not revealed by life expectancy and other measures of average mortality levels (10). For values of life expectancy at birth from under 20 to above 85, lifespan equality rises linearly (Fig 1A). This relationship between life expectancy and lifespan equality has been found to hold in a lifespan continuum over millions of years of primate evolution, in several countries and between subgroups in a population (4-9, 11-13).

This dual advance, however, might be coincidental rather than causal. Doubt is sown by messier relationships between life expectancy and lifespan equality in other datasets and using other indicators (14). The United States, for example, has relatively low equality in lifespans in comparison with other countries that have similar levels of life expectancy (15); Scotland reached similar levels of life expectancy with higher lifespan inequality than England and Wales since 1980 (16); Finnish females from lower educational levels experienced increases in life expectancy while lifespan equality decreased at age 30 since the 1970s (10). In Denmark lifespan equality decreased among the lowest income subgroup over the 1986-2014 period although life expectancy increased (17). In some countries in Eastern Europe and Latin America, life expectancy and lifespan equality moved independently over periods of slow improvements in life expectancy (18-20). Indeed, in many countries and subgroups within a country in recent decades, lifespan equality declined although the average lifespan rose or vice versa (as indicated by the points in the second and fourth quadrants of Fig 1B). In addition, causes of death that contributed to increasing life expectancy somewhat differ from those that increased equality in lifespans in developed countries after 1970 (21, 22). Nonetheless, in spite of these exceptions and discrepancies, life expectancy and lifespan equality generally move in the same direction (9).

In this article, we develop a mathematical framework to explore how life expectancy at birth and lifespan equality relate to each other and evolve over time. We rely on two dimensions of aging: the average length of life (pace) and the relative variation in length of life (shape) (23). The former refers to how fast aging occurs, while the latter describes how sharply populations age. The shape of mortality pertains to the distribution of lifespans. Statisticians and demographers, based on both theoretical and practical considerations, have developed different indicators to summarize the distribution of lifespans (24, 25). Here, we measure average length of life by life expectancy, and we analyze three different indicators of lifespan equality. These indicators are variants of 1) lifetable entropy, 2) the Gini coefficient, and 3) the coefficient of variation of the age-at-death distribution (26, 27). Other indicators of absolute dispersion in lifespans exist, such as the variance of the age-at-death distribution, its standard deviation , or life years lost due to death (28, 29). However, these are pace indicators measured in units of time, and do not capture the dimensionless shape of aging (24).

We focus on how age-specific mortality improvements change lifespan equality and life expectancy at birth. We analyze changes over time on these two longevity measures for Swedish females since the 18th century, and 48 additional populations with reliable data from the Human Mortality Database since the beginning of the 20th century (3).

***Trends in life expectancy and lifespan equality***

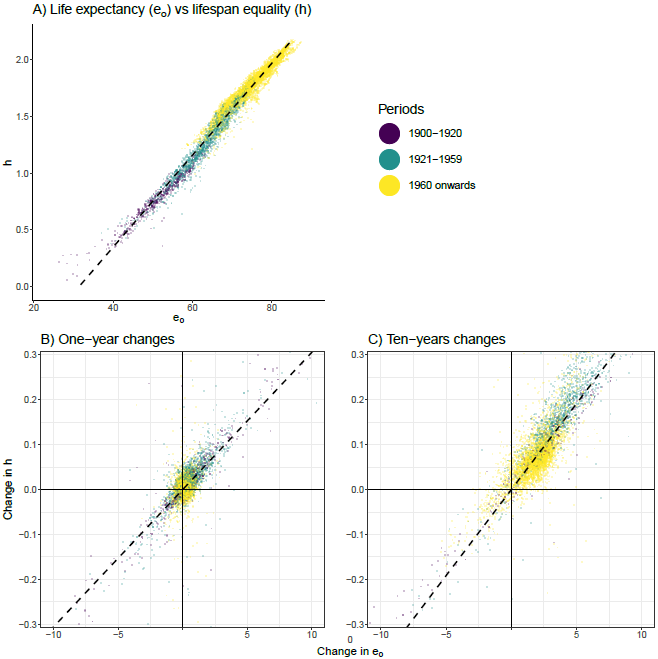
Life expectancy at birth for both men and women increased throughout the 20th century (3, 30). Paralleling the rise of life expectancy, all countries included in our study became more equal in lifespans (Fig. 1A). This is a significant advance in giving people more equitable opportunities. Furthermore, the rise in lifespan equality has influenced the decisions individuals make over their life course, such as when to have children, study, work or retire, because such decisions are based not only on expected lifetime but also on uncertainty about age at death (12). Analysis of the relationship between life expectancy at birth and lifespan equality, as measured by , a log-transformation of lifetable entropy (see Material and Methods and Box 1), indicates a strong correlation (Pearson coefficient of 0.985 for the data in Fig. 1A). We also analyzed the relationship between average lifespan and two other measures of lifespan equality based on the Gini coefficient and the coefficient of variation and found similarly high correlations, .981 and .975 respectively (Supplemental Material [SM], Fig. S1). Although life expectancy and lifespan equality have been positively correlated, it is apparent that the relationship is less strong and often reversed in recent

Figure 1 Panel A. Association between life expectancy at birth and lifespan equality . Panel B. Association between changes in life expectancy and lifespan equality. Panel C. Association between changes over ten-year periods in life expectancy and lifespan equality.

decades, resulting in negative correlations in some countries in yearly and ten-years changes (Fig. 1B-C).

**How strong is the relationship between life expectancy and lifespan equality over time?**

To study how strongly life expectancy and lifespan equality are related over time and whether they respond in the same direction to age-specific mortality changes, we complement demographic analysis with time series analysis (see SM section A. for details). This framework is designed to integrate the stochastic properties of dynamics over time (6, 7). Focusing on changes over time improved our analysis by avoiding misleading inferences from correlations, such as confounding due to unobserved or unmeasured variables (31). Econometric time-series theory indicates that life expectancy and lifespan equality have a long-run relationship if there exists a single process that drives both indicators towards a long-term equilibrium, even if temporary departures from it occur (as observed more often in recent decades). If this equilibrium exists, the changes over time in lifespan equality are proportional to the changes in life expectancy in the long term. In other words, while life expectancy and lifespan equality increase over time, a linear combination of both leads to a residual time series consistent with stationarity (i.e., with stable mean and variance), referred to as cointegration in time series analysis (SM section A.1.2).

The results reveal that in most populations, life expectancy and lifespan equality are linked by a long-run relationship for both sexes (see SM Fig. S2). In 91% of the populations we investigated (males and females from 45 countries and regions by sex) this relationship holds under the same model specifications (SM section A.2); similar results are exhibited for all three indicators of lifespan equality (SM Fig. S2). At the 5% significance level, negative results are expected for 5% of the cases due to random variations. We got negative results in 9% cases. So, the importance of negative results in specific populations should not be overly emphasized (see SM section A.3). These results hold for countries that have experienced substantially different mortality patterns, including women in Japan; men in the U.S. with life expectancy of about 77 years and relatively high lifespan inequality (15); and men in Russia and Ukraine with the lowest levels in life expectancy in this study (about 65 and 66 years in 2013, respectively) and high inequality (19). Importantly, for every population in our study, females’ lives tend to be longer and more equal compared to males in a given year, consistent with previous research (9, 32). This underscores the advantage of females over males not only in average lifespan but also in lower uncertainty about age at death.

**Age-specific dynamics of mortality**

Changes in life expectancy and in lifespan equality over time are weighted averages of rates of progress in reducing age-specific mortality, albeit with different weights (Material and Methods). These weights, for life expectancy and the product for lifespan equality, evolve over time and vary by age. They indicate the potential gain (loss) in life expectancy and lifespan equality if lives are saved at a specific age and in a given period. Panels A and B in Fig. 2 show the corresponding weights for life expectancy, at birth and from age 5 for Swedish women. From the 18th to the first part of 20th century, the largest potential increases in life expectancy were concentrated in infant mortality. The effect on life expectancy improvements due to saving lives in midlife was higher than at older ages. This changed dramatically after 1950, when the effect of infant mortality decreased significantly. By 2010, the effect of reducing mortality by 1% at birth was the same as reducing mortality by 1% at age 71. In the 21st century, saving lives between ages 5 and 40 years has a negligible effect on life expectancy, as opposed to the relatively high impact of these ages before 1900. A shift towards the importance of older ages is clear over time. This ongoing wave towards older ages is in line with recent evidence documenting an advancing front of old-age survival that has driven recent increases in average lifespan (33). Indeed, the postponement of old-age mortality is an ongoing process that started more than 50 years ago (34, 35). Panels A and B in Fig. 2 show that whenever mortality improvements occur life expectancy increases. The size of the increase depends on the ages at which lives are saved. These improvements and the weights are the drivers of life expectancy at birth over time.

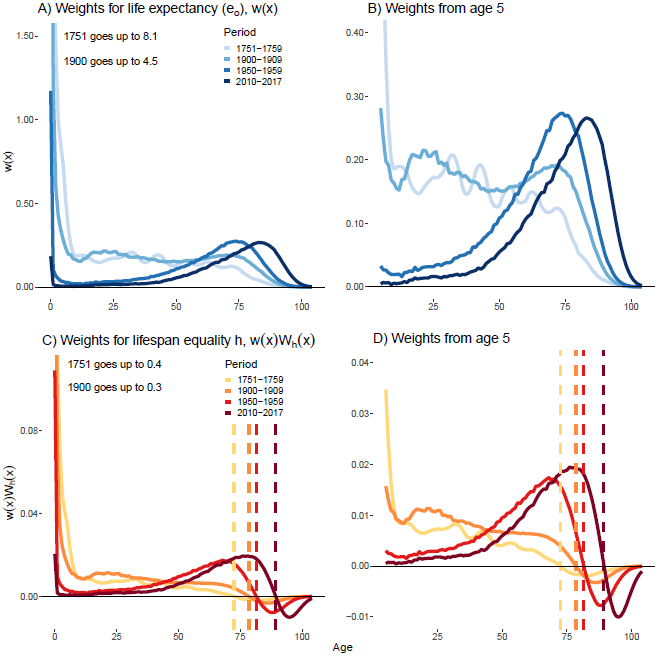


Figure 2Weights for the changes in life expectancy (panels A and B) and lifespan equality (panels C and D). Each line refers to a given period, and represent how life expectancy and lifespan equality react to age-specific mortality improvements

Panels C and D in Fig 2 show the weights, , for lifespan equality . Like panels A and B, each value represents the effect of reducing mortality at a given age would have on lifespan equality. Saving lives at very young ages had the largest effect on increasing equality of lifespans throughout the 18th, 19th and 20th centuries. In contemporary Sweden, the impact of reducing mortality at birth on lifespan equality is the same as saving lives at ages between 76 and 80 years. As with life expectancy, there is an ongoing shift towards older ages, but with an important difference. At older ages, there is a threshold age above which saving lives decreases lifespan equality (Box 1). This age is depicted with the dashed lines colored according to the respective period: an increase of this age over time clearly appears in the graphs. The threshold age gives valuable information for understanding of the relationship between life expectancy at birth and lifespan equality: to the extent that improvements at ages below the threshold age outpace those above it, life expectancy will move in the same direction as lifespan equality (36). The shift from positive to negative effects has previously been explored using other indicators (37, 38). The three lifespan equality indicators that we analyze behave similarly (see SM, Fig. S3); their sensitivity to changes in age-specific mortality resembles that of other indices of lifespan variation (25).

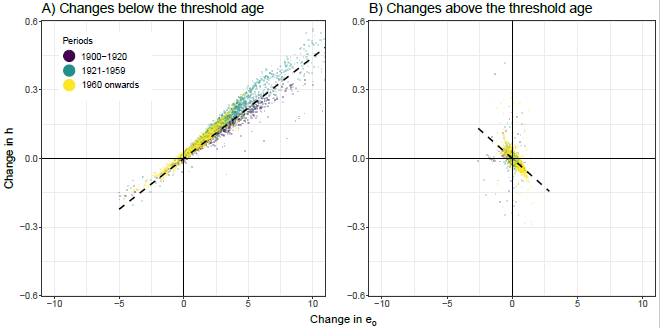


Figure 3. Panel A . Association between 10-year changes in life expectancy at birth and lifespan equality below the threshold age. Panel B. Association between 10-year changes in and above the threshold age. Dotted lines show the directions of the relationship below and above the threshold age.

Fig. 3A shows the contributions, in years, of mortality fluctuations below the threshold age (early-component), and Fig. 3B contributions above the threshold age (late-component) to changes in life expectancy and lifespan equality in 10-year rolling periods for all countries included in our study. The points in the first and third quadrants reflect a mix of reductions in death rates at some ages below and above the threshold and increases at other ages: because the weights for specific ages differ for life expectancy and lifespan equality, the aggregate effect of such a mix of mortality changes can be positive (negative) for life expectancy and negative (positive) for lifespan equality. The sum of the early and late components gives the total change in each indicator (Fig 1C). We report similar results for the two other indicators of lifespan equality in SM Fig. S4. There is a strong positive association between changes in life expectancy and lifespan equality below the threshold age, while the relationship is negative above that threshold. Since the two effects oppose each other, as shown by the regression lines, the relationship is driven by the component that makes the larger contribution. Reductions in death rates below the threshold age were significantly larger than reductions above it before 1960, resulting in a strong positive association between life expectancy and lifespan equality. Since 1960, mortality reductions above the threshold age have become more comparable in magnitude to the early-life component, with more increases in life expectancycoinciding with decreases in lifespan equality. Until now, the absolute change in both indicators per decade is mainly driven by mortality changes below the threshold age (83.7% and 82.0% on average per decade for lifespan equality and life expectancy, respectively [Fig. 3 and Box 1 panel C]).

As life expectancy increases, the threshold age also increases (Box 1 and Fig S5). There is then more scope to save early lives below the threshold age and maintain the positive relationship between life expectancy and lifespan equality. This is an essential characteristic of the long-run equilibrium. Progress, however, after the threshold age has been increasing. For example, in Sweden the most common age at death at older ages was stagnant up until the 1950s when it started rising with life expectancy (Box 1 panel A), and contributions to changes in life expectancy and lifespan equality increased above the threshold age (Box 1 panel C). These results underscore the effect of mortality improvements at advanced ages (i.e. above the threshold age) in recent years and shed light on recent interruptions in the relationship between changes in life expectancy and lifespan equality. This process follows a redistribution of mortality over age and causes of death (21, 39, 40). In the past, deaths were concentrated at young and working ages, mainly due to infectious diseases and to some extent wars and famines that resulted in high inequality of lifespans (41). Nowadays, thanks to major medical improvements, lifesaving is concentrated at older ages, sometimes above the threshold age.

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**Box 1. The threshold age and life expectancy**

Lifespan equality, measured by , refers to an indicator closely related to lifetable entropy, which was first developed by Leser (27) and further explored by Keyfitz and Golini (42-44) and Demetrius (42-44). Lifetable entropy is a dimensionless indicator of the relative variation in the length of life compared to life expectancy at birth, and can be expressed as

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The function denotes the probability of surviving from birth to age , refers to life disparity or the average remaining life expectancy at age at death (45-47), and is life expectancy at birth.

Lifespan equality measured by has previously been used as an indicator of lifespan equality (9). If mortality improvements over time occur at all ages, there exists a unique threshold age that separates positive from negative contributions to (36). Because is a logarithmic transformation of , it has the same threshold age, which we denote by (dashed line in Fig. 2). This threshold is reached when

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where is the cumulative hazard to age and is the lifetable entropy conditioned on surviving to age (36).

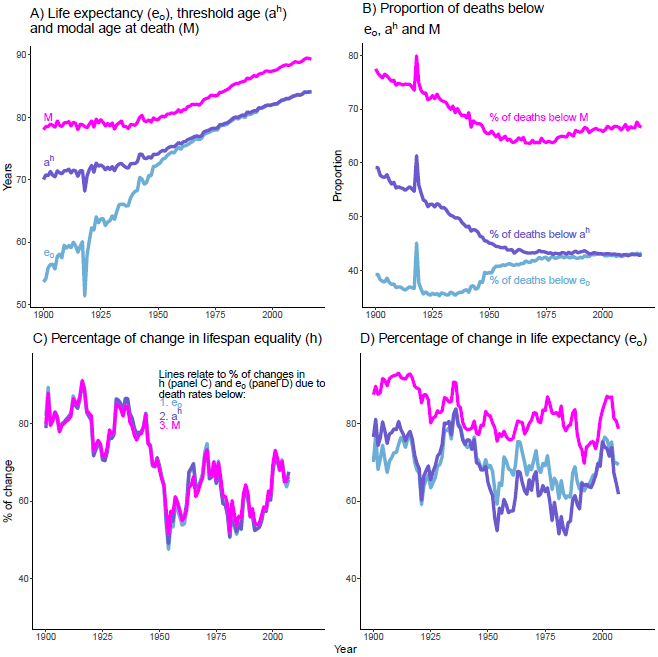
Fig. B1 shows the evolution of life expectancy, the threshold age and the most common age at death after infancy for Swedish females since 1900 (Panel A). The figure highlights how the two measures move together, and that the gap between them has radically declined in recent decades. The threshold age (Panel A) is the age that separates “early” from “late” deaths in terms of their effect on lifespan equality for Swedish females. In other words, the threshold age is the age at which mortality improvements do not change lifespan equality. Averting deaths before this age increases equality, while averting deaths after that age has the opposite effect. It is a population-specific measure, which depends on the observed mortality pattern, and it is a unique age that generally is higher than life expectancy. The two measures move in the same direction, either increasing or decreasing together. The modal age at death was constant before 1950 and rose in tandem with life expectancy and lifespan equality thereafter. More than 40% of deaths occur below life expectancy and the threshold age, while more than 60% of deaths occur below the most common age at death (Panel B). Panel C shows that mortality improvements below life expectancy and the threshold age were responsible for around 80% of gains in life expectancy and lifespan equality in the beginning of the 20th century, while they are responsible for around 60% in contemporary Sweden.

Figure B1 Life expectancy at birth , lifespan equality and the modal age at death M for Swedish females 1900—2017.

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**Outlook**

The dynamics of life expectancy and of lifespan equality are driven by changes in age-specific death rates. The impact of the change at some age differs somewhat for the two measures. At younger ages, the impacts are similar. After a threshold age late in life a reduction in age-specific death rates increases life expectancy but decreases lifespan equality. Because of progress in recent decades in reducing death rates above the threshold age, rises in life expectancyincreasingly coincide with declines in lifespan equality. For the populations we analyzed, in the period 1900-1950, less than 16% of annual changes in average lifespan coincided with opposite changes in lifespan equality. In the decade 1960 this discrepancy rose to 47%; and thereafter the average has been around 32%. Are there paths other than the joint, linear rise in Fig. 1A that might have been followed if social conditions and public policies had been different? This is an intriguing question that can be examined in our framework.

Fig. 4 shows the relationship between life expectancy and lifespan equality for Swedish women from 1751 to 2017 under three different scenarios. Blue triangles refer to observed life expectancy from values below age 20 in 1773 to age 84.1 in 2017. The process of increasing life expectancy with greater equality in individual lifespans has been referred to as the compression of mortality or the rectangularization of survivorship, and has been studied from various perspectives in the last couple of decades (5-9, 19, 48). Understanding the dynamics of the compression of mortality is important for forecasting heterogeneity in future age-patterns of population health as well as for assessments of the timing of individual mortality (10).

Consider the difference of life expectancy and lifespan equality between two consecutive years. The regression line in Fig. 4 indicates that on average the life expectancy change is about 25.4 times the lifespan equality change, a value close to the 27 reported elsewhere (9). Here, we demonstrate that each of these first differences, as an approximation to the time derivative (Material and Methods), is a weighted total of mortality improvements in a given year (Fig. 2). Our main motivation lies on the remarkably tight relationship between life expectancy and lifespan equality through time illustrated by the regression line (slope 0.04, p<.001). For example, in 1773, Sweden underwent the last major famine that caused starvation across the country (49). Approximately 50% of excess deaths were due to dysentery, and most deaths (20%) were concentrated in infancy (41). Even under periods of such mortality stress, observed life expectancy and lifespan equality fall on line that holds in more favorable years. Is this tight connection coincidental or a result of fundamental social and physiological forces? We have shown that the connection is largely due to reductions in death rates at younger ages. Can more be said?

The observed path (blue points Fig. 4) is a combination of age-specific mortality improvements and the weights shown in Fig. 2. Improvements in mortality are uneven across ages (45). Hence we explored an alternative scenario in which the same rate of mortality reduction (or increase) occurred at all ages, with the rate chosen to be consistent with observed levels of life expectancy over time. The red rhombuses in Fig. 4 illustrate the resulting trajectory for Sweden. When the average lifespan rises above 40 years, levels of lifespan equality start to diverge and become lower than the observed ones. The relationship between life expectancy becomes non-linear and levels off at around a life expectancy at birth of 70 years.

Another hypothetical scenario is represented by the purple squares labelled ``optimal equality''. This curve refers to the case when all progress in reducing death rates is concentrated at the youngest ages, when progress most increases lifespan equality. For example, to get the 1752 life expectancy level from 1751, only deaths at age zero are reduced. Then when deaths at birth are zero, deaths are reduced at age 1, then age 2, and so on, to match the observed life expectancy in the following years. That is, all lifesaving is concentrated at the youngest age(s) at which deaths still occur. Results yield a steeper slope (0.051, p<.001), which translates into larger equality in individual lifespans at levels of life expectancy after age 50.

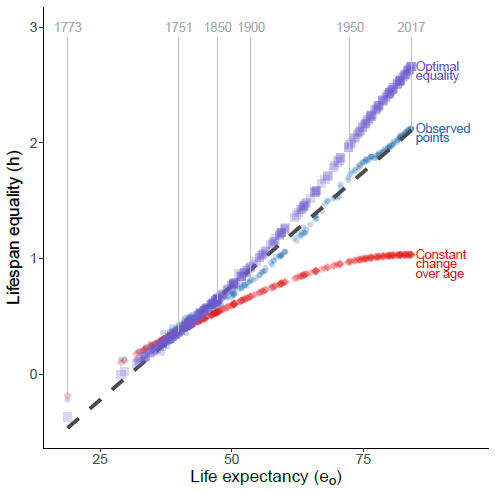


Figure 4 Life expectancy at birth and lifespan equality for three different scenarios: 1) Observed points: Swedish females 1751-2016; 2) Optimal equality: lifespan equality derived by matching observed life expectancy levels by reducing the youngest age; and 3) Constant change in mortality improvements over age matching observed life expectancy levels.

Consider now a third scenario. In every year, age-specific rates of improvement in death rates are chosen such that (1) life expectancy changes as observed and (2) lifespan equality in-creases (or decreases) optimally. That is, when life expectancy increases, progress is concentrated at the ages when change in death rates most increases lifespan equality. If life expectancy de-creases, then declines in age-specific death rates are concentrated at the ages that have the least effect on lifespan equality. Fig. # shows the change over time for Swedish females in lifespan equality under this scenario as well as the observed change in lifespan equality. In most years the observed change is more than 50% of the optimal change.

These three alternative scenarios show that the narrow passageway that describes the relationship between life expectancy at birth and lifespan equality is not a coincidence. The transition from low levels of average lifespan and high variation in length of life to longer and more equal lifespans is a result of saving lives at ages that matter—but not quite optimally. The tight link between life expectancy and lifespan equality has been shaped by improvements in mortality at the ages most important for life expectancy and for lifespan equality, early ages in the 18th century and adult ages today.

In recent years, however, more instances of a temporary reversal of the relationship between life expectancy and lifespan equality have been observed in several countries and subgroups of populations (10, 18-20). Often these cases were due to midlife mortality deterioration or to major improvements in old-age mortality above the threshold age. In Sweden, death rates among octogenarians and nonagenarians have fallen since 1950 (50). For other developed countries, the pattern has been similar (51). If improvements at advanced ages continue and if they outpace those made at younger ages, the pattern of the relationship between life expectancy and lifespan equality could reverse in the future. It is, however, unlikely that rates of improvement above the threshold age will outpace progress at younger ages in the long term. Furthermore, as life expectancy increases, the threshold age will increase.

Across primate species, there is a rough association of life expectancy and lifespan equality. Several instances, however, where a relationship between the pace and shape of aging is not found have been documented in other species. Across the tree of life, 46 diverse species did not show a strong correlation between life expectancy and lifespan equality (52), and among plants a nonlinear, but weak, positive association has been reported (53). These findings compare different species, whereas our results are for a single species in a changing environment. One study, of the nematode worm *C. elegans* and other in*Drosophila melanogaster,* held under different conditions found that lifespan equality appeared to be independent of life expectancy (54, 55).

For humans, a sharp worsening of conditions tends to lead to substantial increases in infant and child mortality (41), lowering both life expectancy and lifespan equality. On the other hand, improvements in standards of living, nutrition, education, public health and other environmental conditions tend, at least when life expectancy is less than 70, to predominately affect life expectancy—and lifespan equality—through reductions in death rates at young ages (2). A key question is whether changes in environmental conditions have their biggest effects on mortality in infancy and younger ages because of human agency and because of human physiology. Do societies act to focus mortality improvements at the ages that most matter or is human mortality for physiological reasons most sensitive at younger ages to environmental changes? Study of the impact of environmental change on life expectancy and lifespan equality in nonhuman primate species, being undertaken by Fernando Colchero, Susan Alberts and colleagues, could shed light on the role of agency versus physiology. More generally, our findings—coupled with the mathematical relationships we derived to analyze how changes in age-specific death rates affect life expectancy and lifespan equality—suggest that a link may be found for species for which environmental change affects life expectancy largely because of changes in death rates at young ages.

**Materials and Methods**

**Data.** We used death rates by age and sex from the Human Mortality Database (3) for 49 countries and regions available from the beginning of the 20th century to the most recent year available in the dataset (7,717 lifetables, see SM Table S1 for detailed information). We constructed lifetables following standard demographic procedures (56). For each population, we investigated life expectancy at birth and lifespan equality by sex. The analysis is restricted to countries with data available for consecutive years (without gaps in the information over time) in order to study age-specific mortality patterns on a yearly basis. We decided not to analyze dispersion at death conditioned on survival to any older age because of major improvements made in early ages during the 20th century (57). In addition, we did not include Chile, Korea and Croatia in the cointegration analysis due to limited data availability, spanning less than 20 years.

**Contributions to mathematical demography**

***Changes over time in life expectancy.*** Changes over time in life expectancy at birth are a weighted average of rates of progress in reducing mortality (29). Let be the period lifetable probability at time of surviving from birth to age , life expectancy at birth can be expressed as

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Because where is the force of morality (hazard rate) at age at time , changes over time in are given by

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A dot over a function denotes its partial derivative with respect to time. For simplicity, variable will be omitted as an argument in the following. We define

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as the age-specific rates of mortality improvement over time and the remaining life expectancy at age , respectively. Then, Eq. (1) can be expressed in terms of these two functions as

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This last result shows that changes over time in life expectancy at birth are a weighted total of rates of progress in reducing mortality, with weights given by the function , as shown by Vaupel and Canudas-Romo (29).

***Measures of lifespan equality and their change over time.*** Several indicators have been proposed to measure variation in age at death (25, 58, 59). Selecting the best measure when comparing aging patterns among populations that differ in length of life is of great importance since indicators vary in their sensitivity to mortality fluctuations and in their mathematical interpretation (25). In this study, we use three indicators based on the pace and shape of aging framework (23), which suggests a set of properties that indicators should satisfy (24, 60).

*A variant of the lifetable entropy:* . A measure of lifespan inequality is the lifetable entropy (27, 42, 44), usually defined as

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where is the lifetable age composition, and is the cumulative hazard to age . Hence, can be interpreted as an average value of the cumulative hazard. It can also be expressed as

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where accounts for the *life* *disparity* or the average number of life-years lost as a result of death (7). For instance, an individual dying at age 50 in a population with remaining life expectancy at age 50 of 20 years would have lost those 20 years of life.

This definition of entropy provides a dimensionless indicator of relative variation in the length of life compared to life expectancy at birth, permitting comparison of populations with different age-at-death distributions (24). An alternative measure to is

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which has previously been used to study lifespan equality across different primate populations, including humans (9). Note that can be interpreted as an indicator of *lifespan inequality,* given that higher values represent more variation in lifespans, whereas  (the logarithm of the inverse) is a measure of *lifespan equality.* From Eq. (3), the variation over time in is given by

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An equivalent expression to Eq. (4) was previously derived using calculus of variation by Fernández and Beltrán-Sánchez (61), who found that

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This shows that changes over time in are equal to minus the relative change in the lifetable entropy . Similarly to life expectancy at birth, Aburto and colleagues (36) proved that

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where are the same weights for changes over time in defined in Eq. (2), and

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Function is the entropy conditioned on surviving to age , where refers to life disparity above age , and is the remaining life expectancy at age (36). Because , it follows that

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with . This result shows that changes in lifespan equality over time are weighted total of rates of progress in reducing mortality , with weights given by the product .

*A variant of the Gini coefficient:* . The Gini coefficient is a popular index in social science used to measure distributions of positive variables, such as income (62). It has also been used to describe inequality in lifespans as a measure of population health and in survival analysis as an indicator of concentration in survival times (24, 26, 63-65). In lifetable notation, the Gini coefficient is given by

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Function relates to perturbation theory as it measures life expectancy from doubling the risk of death at all ages. From Eq. (6), can also be expressed in terms of the lifetable age distribution,

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Note that is a dimensionless indicator of lifespan equality, bounded between 0 and 1. If lifespans are completely concentrated, all individuals die at the same age, the indicator equals 1; if they are equally spread the indicator tends to 0. In addition, if two babies are born at the same time in a population, then measures their shared lifespan as a proportion of life expectancy (66). An alternative indicator to the Gini coefficient is the logarithm of its inverse

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| --- | --- |
|  | [7] |

which is also a measure of equality rather than inequality. Note that the derivative of with respect to time is

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| --- | --- |
|  |  |

Hence, changes over time in are given by

|  |  |
| --- | --- |
|  | [8] |

Similar to and , the time derivative of can be re-expressed as

|  |  |
| --- | --- |
|  | [9] |

where are the same weights for changes over time in, and

|  |  |
| --- | --- |
|  |  |

Function is defined as

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| --- | --- |
|  |  |

and can be interpreted as lifespan equality above age . A detailed proof of Eq. (9) can be found in SM Section B. This result shows that changes in lifespan equality over time, measured by , are a weighted total of the rates of progress in reducing mortality , with weights given by the product .

*A variant of the coefficient of variation:* . The coefficient of variation of the age-at-death distribution is the quotient between its standard deviation and the life expectancy at birth, say

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| --- | --- |
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This indicator has been previously used to measure lifespan inequality (22, 24). Here, we define a new measure of lifespan equality as the logarithm of the inverse of the coefficient of variation,

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| --- | --- |
|  | [10] |

Similar to the previous cases, changes over time in are given by

|  |  |
| --- | --- |
|  | [11] |

which can be re-expressed as

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| --- | --- |
|  | [12] |

As before, are the weights for , whereas are the additional weights defined as

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| --- | --- |
|  |  |

where

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| --- | --- |
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Note that is a weighted average of deviations from life expectancy at age , which can be expressed as the difference between the average age of the population above age () and the life expectancy at birth. A detailed proof of Eq. (12) can be found in SM Section C. This result shows that changes over time in our alternative measure of the coefficient of variation are a weighted total of the rates of progress in reducing mortality , with weights given by the product.

**Demographic methods to calculate threshold ages and age-specific contributions.** From life tables, we calculated for each of the three indicators the threshold age below which averting deaths increases lifespan equality, and above which equality decreases. These threshold ages, which we denote by , and respectively, are reached when the corresponding weights equal 0; that is, when , or . This follows from Eq. (5), (9) and (12), and the fact that and are strictly positive functions (the latter in the assumption that mortality improves at all ages).

We quantified age-specific contributions to yearly changes in life expectancy and lifespan equality for all the data available and estimated contributions above and below those thresholds. We used a model defined on a continuous framework that assumes gradual change in the covariates over time (67) used in previous studies of lifespan inequality (11, 18, 19, 22).

**Stochastic properties of life expectancy and lifespan equality.** We analyzed the stochastic properties of and lifespan equality over time to determine whether they are stationary processes (for further details see SM Section A). In case of non-stationarity, we also find the order of integration. We performed the Kwiatkowski–Phillips–Schmidt–Shin test (KPSS) (68) for  and lifespan equality and the augmented Dickey–Fuller test (ADF) (69) in their levels and first differences, respectively (we also perform tests against higher orders of integration but could not reject the hypothesis that the variables were integrated at a lower level.). Using the 95% critical values, the null hypothesis of stationarity can be rejected in 94.9% of the cases for life expectancy and 93.9% for lifespan equality (). Moreover, at the same level, the null-hypothesis of a unit-root in their first differences is rejected 97% for and lifespan equality. These analyses suggest that the variables are nonstationary processes and achieve stationarity after differencing once for both females and males. In the statistical analysis we treat both variables as integrated of order one. The concept of cointegration was developed to avoid misleading interpretations regarding the relationship between two integrated variables (70). It refers to the case of a model that can adjust for stochastic trends to produce stationary residuals, and it allows detecting stable long-run relationships among integrated variables. Formally, two cointegrated variables can be expressed using a 2-dimensional vector autoregressive model in its equilibrium correction (VECM) form, defined as

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|  | [14] |

Operator denotes the first differences; is a vector of stochastic variables(and lifespan equality in our case); and are vectors of full rank; is a vector of constants; and is a vector of normally, independently, and identically distributed errors with zero means and constant variances. We specify the model with an unrestricted constant in the cointegration space and dummy variables in contexts where life expectancy experienced historical shocks, such as World Wars and epidemics (see SM table S2 and Section A for more details and sensitivity analyses).

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**Author contributions:**

JMA and JWV designed the research. UB and SK suggested the cointegration framework and helped with interpretation and data analysis. JMA and JWV suggested the demographic methodology. FV, JMA and JWV developed the mathematical demography framework. All authors interpreted results. JMA, FV, UB and JWV wrote the manuscript with input from SK.

**Figure legends**

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